2.2 Analyze Conditional Statements

Before You used definitions.

Now You will write definitions as conditional statements.

Why? So you can verify statements, as in Example 2.

Key Vocabulary

- **conditional statement** converse, inverse, contrapositive
- **if-then form** hypothesis, conclusion
- **negation**
- **equivalent statements**
- **perpendicular lines**
- **biconditional statement**

A **conditional statement** is a logical statement that has two parts, a *hypothesis* and a *conclusion*. When a conditional statement is written in **if-then form**, the "if" part contains the **hypothesis** and the "then" part contains the **conclusion**. Here is an example:

If **it is raining**, then **there are clouds in the sky**.

Hypothesis — Conclusion

EXAMPLE 1 Rewrite a statement in if-then form

Rewrite the conditional statement in if-then form.

a. All birds have feathers.

b. Two angles are supplementary if they are a linear pair.

Solution

First, identify the **hypothesis** and the **conclusion**. When you rewrite the statement in if-then form, you may need to reword the hypothesis or conclusion.

a. **All birds** have **feathers**.

If **an animal is a bird**, then **it has feathers**.

b. **Two angles are supplementary** if **they are a linear pair**.

If **two angles are a linear pair**, then **they are supplementary**.

✓ GUIDED PRACTICE for Example 1

Rewrite the conditional statement in if-then form.

1. All 90° angles are right angles.

2. $2x + 7 = 1$, because $x = -3$.

3. When $n = 9$, $n^2 = 81$.

4. Tourists at the Alamo are in Texas.

NEGATION The **negation** of a statement is the *opposite* of the original statement. Notice that Statement 2 is already negative, so its negation is positive.

Statement 1 The ball is red.

Negation 1 The ball is *not* red.

Statement 2 The cat is *not* black.

Negation 2 The cat is black.

VERIFYING STATEMENTS Conditional statements can be true or false. To show that a conditional statement is true, you must prove that the conclusion is true every time the hypothesis is true. To show that a conditional statement is false, you need to give *only one* counterexample.

RELATED CONDITIONALS To write the **converse** of a conditional statement, exchange the **hypothesis** and **conclusion**.

READ VOCABULARY
To *negate* part of a conditional statement, you write its negation.

To write the **inverse** of a conditional statement, negate both the hypothesis and the conclusion. To write the **contrapositive**, first write the converse and then negate both the hypothesis and the conclusion.

Conditional statement If $m\angle A = 99°$, then $\angle A$ is obtuse.	
Converse If $\angle A$ is obtuse, then $m\angle A = 99°$.	both false
Inverse If $m\angle A \neq 99°$, then $\angle A$ is not obtuse.	
Contrapositive If $\angle A$ is not obtuse, then $m\angle A \neq 99°$.	both true

EXAMPLE 2 Write four related conditional statements

Write the if-then form, the converse, the inverse, and the contrapositive of the conditional statement "Guitar players are musicians." Decide whether each statement is *true* or *false*.

Solution

If-then form If you are a guitar player, then you are a musician. *True*, guitars players are musicians.

Converse If you are a musician, then you are a guitar player. *False*, not all musicians play the guitar.

Inverse If you are not a guitar player, then you are not a musician. *False*, even if you don't play a guitar, you can still be a musician.

Contrapositive If you are not a musician, then you are not a guitar player. *True*, a person who is not a musician cannot be a guitar player.

GUIDED PRACTICE for Example 2

Write the converse, the inverse, and the contrapositive of the conditional statement. Tell whether each statement is *true* or *false*.

5. If a dog is a Great Dane, then it is large.

6. If a polygon is equilateral, then the polygon is regular.

EQUIVALENT STATEMENTS A conditional statement and its contrapositive are either both true or both false. Similarly, the converse and inverse of a conditional statement are either both true or both false. Pairs of statements such as these are called *equivalent statements*. In general, when two statements are both true or both false, they are called **equivalent statements**.

DEFINITIONS You can write a definition as a conditional statement in if-then form or as its converse. Both the conditional statement and its converse are true. For example, consider the definition of *perpendicular lines*.

READ DIAGRAMS
In a diagram, a red square may be used to indicate a right angle or that two intersecting lines are perpendicular.

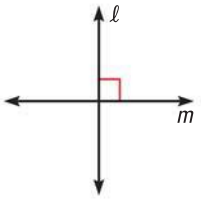
KEY CONCEPT *For Your Notebook*

Perpendicular Lines

Definition If two lines intersect to form a right angle, then they are **perpendicular lines**.

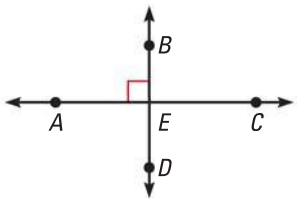
The definition can also be written using the converse: If two lines are perpendicular lines, then they intersect to form a right angle.

You can write "line ℓ is perpendicular to line m" as $\ell \perp m$.



EXAMPLE 3 Use definitions

Decide whether each statement about the diagram is true. Explain your answer using the definitions you have learned.



a. $\overleftrightarrow{AC} \perp \overleftrightarrow{BD}$

b. $\angle AEB$ and $\angle CEB$ are a linear pair.

c. $\overrightarrow{EA}$ and $\overrightarrow{EB}$ are opposite rays.

Solution

a. This statement is *true*. The right angle symbol in the diagram indicates that the lines intersect to form a right angle. So you can say the lines are perpendicular.

b. This statement is *true*. By definition, if the noncommon sides of adjacent angles are opposite rays, then the angles are a linear pair. Because $\overrightarrow{EA}$ and $\overrightarrow{EC}$ are opposite rays, $\angle AEB$ and $\angle CEB$ are a linear pair.

c. This statement is *false*. Point E does not lie on the same line as A and B, so the rays are not opposite rays.

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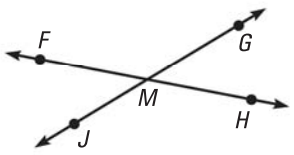
GUIDED PRACTICE for Example 3

Use the diagram shown. Decide whether each statement is true. *Explain* your answer using the definitions you have learned.



7. $\angle JMF$ and $\angle FMG$ are supplementary.

8. Point M is the midpoint of $\overline{FH}$.

9. $\angle JMF$ and $\angle HMG$ are vertical angles.

10. $\overleftrightarrow{FH} \perp \overleftrightarrow{JG}$

READ DEFINITIONS
All definitions can be interpreted forward and backward in this way.

BICONDITIONAL STATEMENTS When a conditional statement and its converse are both true, you can write them as a single *biconditional statement*. A **biconditional statement** is a statement that contains the phrase "if and only if."

Any valid definition can be written as a biconditional statement.

EXAMPLE 4 Write a biconditional

Write the definition of perpendicular lines as a biconditional.

Solution

Definition If two lines intersect to form a right angle, then they are perpendicular.
Converse If two lines are perpendicular, then they intersect to form a right angle.
Biconditional Two lines are perpendicular if and only if they intersect to form a right angle.

GUIDED PRACTICE for Example 4

11. Rewrite the definition of *right angle* as a biconditional statement.

12. Rewrite the statements as a biconditional.

If Mary is in theater class, she will be in the fall play. If Mary is in the fall play, she must be taking theater class.

2.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 11, 17, and 33

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 25, 29, 33, 34, and 35

SKILL PRACTICE

1. VOCABULARY Copy and complete: The __?__ of a conditional statement is found by switching the hypothesis and the conclusion.

2. ★ WRITING Write a definition for the term *collinear points*, and show how the definition can be interpreted as a biconditional.

EXAMPLE 1
on p. 79
for Exs. 3–6

REWRITING STATEMENTS **Rewrite the conditional statement in if-then form.**

3. When $x = 6$, $x^2 = 36$.

4. The measure of a straight angle is 180°.

5. Only people who are registered are allowed to vote.

6. ERROR ANALYSIS *Describe* and correct the error in writing the if-then statement.

Given statement: All high school students take four English courses.

If-then statement: If a high school student takes four courses, then all four are English courses.

EXAMPLE 2 on p. 80 for Exs. 7–15

WRITING RELATED STATEMENTS **For the given statement, write the if-then form, the converse, the inverse, and the contrapositive.**

7. The complementary angles add to 90°.

8. Ants are insects.

9. $3x + 10 = 16$, because $x = 2$.

10. A midpoint bisects a segment.

ANALYZING STATEMENTS **Decide whether the statement is *true* or *false*. If false, provide a counterexample.**

11. If a polygon has five sides, then it is a regular pentagon.

12. If $m\angle A$ is 85°, then the measure of the complement of $\angle A$ is 5°.

13. Supplementary angles are always linear pairs.

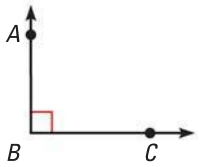
14. If a number is an integer, then it is rational.

15. If a number is a real number, then it is irrational.

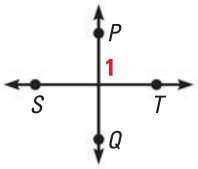
EXAMPLE 3 on p. 81 for Exs. 16–18

USING DEFINITIONS **Decide whether each statement about the diagram is true. *Explain* your answer using the definitions you have learned.**

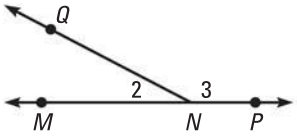
16. $m\angle ABC = 90°$



17. $\overleftrightarrow{PQ} \perp \overleftrightarrow{ST}$



18. $m\angle 2 + m\angle 3 = 180°$



EXAMPLE 4 on p. 82 for Exs. 19–21

REWRITING STATEMENTS **In Exercises 19–21, rewrite the definition as a biconditional statement.**

19. An angle with a measure between 90° and 180° is called *obtuse*.

20. Two angles are a *linear pair* if they are adjacent angles whose noncommon sides are opposite rays.

21. *Coplanar points* are points that lie in the same plane.

DEFINITIONS **Determine whether the statement is a valid definition.**

22. If two rays are *opposite rays*, then they have a common endpoint.

23. If the sides of a triangle are all the same length, then the triangle is *equilateral*.

24. If an angle is a *right angle*, then its measure is greater than that of an acute angle.

25. ★ **MULTIPLE CHOICE** Which statement has the same meaning as the given statement?

GIVEN ▶ You can go to the movie after you do your homework.

(A) If you do your homework, then you can go to the movie afterwards.

(B) If you do not do your homework, then you can go to the movie afterwards.

(C) If you cannot go to the movie afterwards, then do your homework.

(D) If you are going to the movie afterwards, then do not do your homework.

ALGEBRA **Write the converse of each true statement. Tell whether the converse is true. If false, *explain* why.**

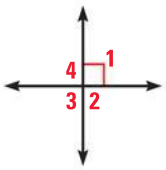
26. If $x > 4$, then $x > 0$.

27. If $x < 6$, then $-x > -6$.

28. If $x \leq -x$, then $x \leq 0$.

29. ★ **OPEN-ENDED MATH** Write a statement that is true but whose converse is false.

30. **CHALLENGE** Write a series of if-then statements that allow you to find the measure of each angle, given that $m\angle 1 = 90°$. Use the definition of linear pairs.



PROBLEM SOLVING

EXAMPLE 1 on p. 82 for Exs. 31–32

In Exercises 31 and 32, use the information about volcanoes to determine whether the biconditional statement is *true* or *false*. If false, provide a counterexample.

VOLCANOES Solid fragments are sometimes ejected from volcanoes during an eruption. The fragments are classified by size, as shown in the table.

31. A fragment is called a *block or bomb* if and only if its diameter is greater than 64 millimeters.

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32. A fragment is called a *lapilli* if and only if its diameter is less than 64 millimeters.

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Type of fragment	Diameter d (millimeters)
Ash	$d < 2$
Lapilli	$2 \leq d \leq 64$
Block or bomb	$d > 64$

33. ★ **SHORT RESPONSE** How can you show that the statement, "If you play a sport, then you wear a helmet." is false? *Explain*.

34. ★ **EXTENDED RESPONSE** You measure the heights of your classmates to get a data set.

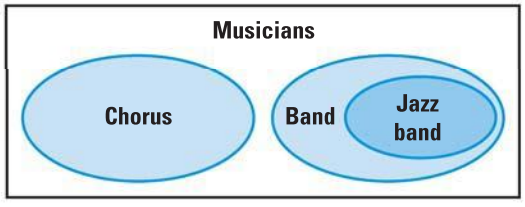
a. Tell whether this statement is true: If x and y are the least and greatest values in your data set, then the mean of the data is between x and y. *Explain* your reasoning.

b. Write the converse of the statement in part (a). Is the converse true? *Explain*.

c. Copy and complete the statement using *mean*, *median*, or *mode* to make a conditional that is true for any data set. *Explain* your reasoning.

Statement If a data set has a mean, a median, and a mode, then the _?_ of the data set will always be one of the measurements.

35. ★ **OPEN-ENDED MATH** The Venn diagram below represents all of the musicians at a high school. Write an if-then statement that describes a relationship between the various groups of musicians.



○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

36. MULTI-STEP PROBLEM The statements below describe three ways that rocks are formed. Use these statements in parts (a)–(c).

Igneous rock is formed from the cooling of molten rock.

Sedimentary rock is formed from pieces of other rocks.

Metamorphic rock is formed by changing temperature, pressure, or chemistry.

a. Write each statement in if-then form.

b. Write the converse of each of the statements in part (a). Is the converse of each statement true? *Explain* your reasoning.

c. Write a true if-then statement about rocks. Is the converse of your statement *true* or *false*? *Explain* your reasoning.

37. ALGEBRA Can the statement, "If $x^2 - 10 = x + 2$, then $x = 4$," be combined with its converse to form a true biconditional?

38. REASONING You are given that the contrapositive of a statement is true. Will that help you determine whether the statement can be written as a true biconditional? *Explain*.

39. CHALLENGE Suppose each of the following statements is true. What can you conclude? *Explain* your answer.

If it is Tuesday, then I have art class.

It is Tuesday.

Each school day, I have either an art class or study hall.

If it is Friday, then I have gym class.

Today, I have either music class or study hall.

MIXED REVIEW

PREVIEW
Prepare for Lesson 2.3 in Exs. 40–45.

Find the product of the integers. *(p. 869)*

40. $(-2)(10)$ **41.** $(15)(-3)$ **42.** $(-12)(-4)$

43. $(-5)(-4)(10)$ **44.** $(-3)(6)(-2)$ **45.** $(-4)(-2)(-5)$

Sketch the figure described. *(p. 2)*

46. $\overleftrightarrow{AB}$ intersects $\overleftrightarrow{CD}$ at point E.

47. $\overleftrightarrow{XY}$ intersects plane P at point Z.

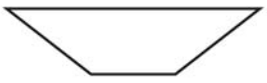
48. $\overleftrightarrow{GH}$ is parallel to $\overleftrightarrow{JK}$.

49. Vertical planes X and Y intersect in $\overleftrightarrow{MN}$.

Find the coordinates of the midpoint of the segment with the given endpoints. *(p. 15)*

50. $A(10, 5)$ and $B(4, 5)$ **51.** $P(4, -1)$ and $Q(-2, 3)$ **52.** $L(2, 2)$ and $N(1, -2)$

Tell whether the figure is a polygon. If it is not, *explain* why. If it is a polygon, tell whether it is *convex* or *concave*. *(p. 42)*

53. 

54. 

55.